

Indian Statistical Institute, Bangalore

B. Math I, First Semester, 2025-26

Mid-semester Examination

Probability I

09.09.25

Maximum Score 60

Duration: 2 Hours

1. (2+2) Suppose $P(A) = 0.3$ and $P(B) = 0.6$.
 - (a) If A and B are disjoint, what is $P(A \cup B)$?
 - (b) If A and B are independent, what is $P(A \cup B)$?
2. (2+2) If n persons, among whom are A and B , stand in a row, what is the probability that there will be exactly r persons between A and B ? If they stand in a ring instead of in a row, what is this probability?
3. (10) A box contains 7 indistinguishable green balls, 5 indistinguishable white balls and 6 indistinguishable black balls. Suppose balls are drawn using simple random sampling with replacement. If n balls are drawn, what is probability that at least one ball of each of the three colours appears in the sample?
4. (8+2+2) Consider a hypergeometric distribution with parameters N, m, n . Here N is the total number of balls among which m are white and the rest are black. A sample of size n is drawn without replacement and the random variable X is the number of white balls in the sample.
 - (a) At what value of the random variable is the pmf (probability mass function) maximum?
 - (b) When $m = 2, n = 3, N = 10$, what are the values of X at which the pmf is positive?
 - (c) With the parameters in (b), where does the pmf attain its maximum?
5. (10) Let Y_1, \dots, Y_n be independent Geometric random variables both parameter p . Show that the distribution of $\sum_{i=1}^n Y_i$ is negative binomial with parameters n and p using the pmf of the distributions and induction.
6. (10) Select 5 balls randomly without replacement from an urn containing balls numbered 1, 2, \dots , 25. Let the random variable X be the largest number on a ball. Find the expectation and variance of X .
7. (10) Players A and B play a series of games. The first player who wins 3 games is declared the winner of the series. Suppose that the chance of player A winning a single game is p , independently of the other games played. Calculate the conditional probability that player A wins the series given that he won the first game.